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SEMILEPTONIC WEAK INTERACTIONS IN NUCLEI: A RELATIVISTIC ANALYSIS

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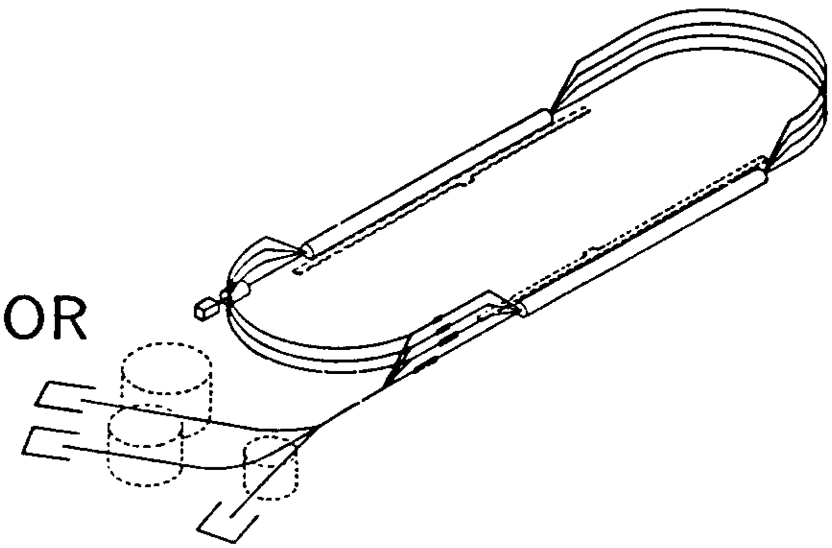

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Semileptonic Weak Interactions in Nuclei: a Relativistic Analysis

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ABSTRACT

We use a consistent relativistic formalism , with relativistic Hartree wavefunctions and a covariant effective current operator with all the properties of the Standard Model, to study semileptonic weak processes in nuclei. Results are obtained for β - decay, μ - capture, charge-changing ν - reactions and neutral ν – scattering involving the isodoublet systems ^{17}F - ^{17}O and ^{3}H - ^{3}He.

The nuclear system is currently described in terms of hadrons or quarks, depending upon the distance scale. Electroweak interactions provide a fundamental tool for studying this system, but a complete kinematic range of probes is necessary to investigate the regions of present theoretical interest. For this purpose the traditional nonrelativistic approach[1] to the analysis of electroweak processes in nuclei is inadequate and a consistent relativistic theoretical framework is needed. QHD (quantum hadrodynamics),[2] a relativistic quantum field theory of the nuclear many-body problem based on mesons and baryons, provides such a framework. It allows us to include relativistic corrections to all orders in closed form, without expanding[3] the conventional matrix elements. A relativistic formalism for electron - nucleus scattering based on QHD in the Hartree approximation was proposed and studied by Serot[4,5] and Walecka[2], and was used to examine high q^2 elastic electron scattering from selected nuclei.[6] In this paper we extend the analysis to include semileptonic weak processes, and apply the formalism to study β-decay, μ^-- capture, charge-changing ν- reactions and neutral ν-scattering processes on the ^{17}F - ^{17}O and ^{3}H - ^{3}He isodoublet systems.

We generate relativistic Hartree wavefunctions[7] by self-consistently solving the coupled meson field equations (for the neutral meson fields ϕ_0, V_0, ρ_0, A_0) and the Dirac equation (for the baryon wavefunctions). The resulting wavefunctions are of the form

$$\psi_{n\kappa m}(\mathbf{x}) = \begin{bmatrix} i[\frac{G_{n\kappa}(r)}{r}]\phi_{\kappa m}(\Omega) \\ -[\frac{F_{n\kappa}(r)}{r}]\phi_{-\kappa m}(\Omega) \end{bmatrix}$$

and the upper and lower radial wavefunctions for ^{17}O and ^{3}He can be obtained.[8]

We use an effective weak current operator[2] together with the Hartree wavefunctions. We assume all formfactors (charge, magnetic, and axial) to have a common momentum transfer dependence $F(q^2) = f_{sn}(q^2)F(0)$, where $f_{sn}(q^2)$ is the empirical dipole fit. From the V - A structure of weak currents and CVC theory, the vector part of the weak charge-changing current can be written in analogy to the electromagnetic effective current operator:

$$\hat{J}_\mu^{(\pm)} = \hat{\bar{\psi}}\gamma_\mu\tau_\pm\hat{\psi} + \frac{(\lambda_p - \lambda_n)}{2M}\frac{\partial}{\partial x_\nu}\left(\hat{\bar{\psi}}\sigma_{\mu\nu}\tau_\pm\hat{\psi}\right).$$

We incorporate the induced pseudo-scalar coupling, $F_P(q^2) = 2MF_A(q^2)/(m_\pi^2 - q^2)$, into the effective axial-vector charge-changing current. This gives :

$$\hat{J}_{\mu 5}^{(\pm)}(x) = \left(\delta_{\mu\nu} + \frac{1}{m_\pi^2 - \partial_\lambda\partial_\lambda}\partial_\mu\partial_\nu\right)\hat{\bar{\psi}}(x)\gamma_\nu\gamma_5 w^{(\pm)}\hat{\psi}(x)$$

where $w^{(}\pm) \equiv F_A(0)\tau_{(\pm)}$. Thus the total charge-changing weak current is given by $\hat{\mathbf{J}}_\mu^{(\pm)} = \hat{J}_\mu^{(\pm)} + \hat{J}_{\mu 5}^{(\pm)}$. The weak neutral current is given by $\hat{\mathbf{J}}_\mu^{(0)} = \hat{\mathbf{J}}_\mu^{V_3} - 2\sin^2\theta_W \hat{J}_\mu^\gamma$, where $\hat{J}_\mu^\gamma \equiv \hat{J}_\mu^S + \hat{J}_\mu^{V_3}$ is the electromagnetic current.

These currents are covariant, correctly describes properties of free nucleons, and contain all the general features of the Standard Model. The vector current is local and conserved in QHD; the axial vector current satisfies PCAC, and the nonlocality reflects the physics of pion-pole dominance.[2]

We must compute the matrix element for a semileptonic process[1] using the covariant effective currents ; this gives some new terms in the matrix element involving $\frac{l \cdot q}{m_\pi^2 + q^2}q_\lambda \times$ (axial-vector multipoles).

The vector and axial vector multipole operators[1] can be written in block 2×2 matrix form:

$$\hat{\mathcal{M}}_{JM} = \hat{M}_{JM} + \hat{m}_{JM}^5 = \begin{pmatrix} QM_J^M & -i\bar{q}\Sigma''^M_J + Q'M_J^M \\ i\bar{q}\Sigma''^M_J + Q'M_J^M & QM_J^M \end{pmatrix}$$

$$\hat{\mathcal{T}}_{JM}^{el} = \hat{T}_{JM}^{el} + \hat{t}_{JM}^{el5} = \begin{pmatrix} \bar{q}\Sigma_J^M + iQ'\Sigma'^M_J & iQ_{(+)}\Sigma'^M_J \\ iQ_{(-)}\Sigma'^M_J & -\bar{q}\Sigma_J^M + iQ'\Sigma'^M_J \end{pmatrix}$$

$$\hat{\mathcal{T}}_{JM}^{mag} = \hat{T}_{JM}^{mag} + \hat{t}_{JM}^{mag5} = \begin{pmatrix} i\bar{q}\Sigma'^M_J + Q'\Sigma_J^M & Q_{(+)}\Sigma_J^M \\ Q_{(-)}\Sigma_J^M & -i\bar{q}\Sigma'^M_J + Q'\Sigma_J^M \end{pmatrix}$$

$$\hat{\mathcal{L}}_{JM} = \hat{L}_{JM} + \hat{l}^5_{JM} = \begin{pmatrix} iQ'\Sigma''^M_J & iQ_{(-)}\Sigma''^M_J \\ iQ_{(+)}\Sigma''^M_J & iQ'\Sigma''^M_J \end{pmatrix}$$

Here we define $Q_{(\pm)} \equiv Q \pm \lambda(E_f - E_i)/2M$, and $\bar{q} \equiv q\lambda/2M$. For charge-changing processes, $Q \equiv \tau_\pm$, $\lambda \equiv (\lambda_p - \lambda_n)\tau_\pm$, and $Q' \equiv \omega_\pm \equiv F_A^{(1)}(0)\tau_\pm$; for neutral-current processes, $Q \equiv (\frac{1}{2} - \sin^2\theta_w)\tau_3 - \sin^2\theta_w$, $\lambda \equiv (\lambda_p - \lambda_n)(\frac{1}{2} - \sin^2\theta_w)\tau_3 - \sin^2\theta_w(\lambda_p + \lambda_n)$, and $Q' \equiv \frac{1}{2}F_A^{(1)}(0)\tau_3$. The single particle multipole operators $(\Sigma^M_J, \Sigma'^M_J, \Sigma''^M_J)$ are the multipole projections of the Pauli matrices, and are defined following the convention of ref. 8.[8]

To compute the cross-sections and rates for the semileptonic processes, we must square the matrix element and evaluate appropriate spin sums. The resulting $\nu-$ (or $\bar{\nu}-$)differential cross-section can be written in terms of q^2 ; also, for isoelastic processes, only even $\hat{M}_J$ and odd $\hat{T}^{mag}_J$, $\hat{T}^{el5}_J$, and $\hat{L}^5_J$ multipoles contribute.[9] Then the charge changing differential cross-section, for isoelastic processes and arbitrary kinematics, becomes:

$$\begin{aligned}
\frac{d\sigma}{dq^2} =& \frac{G^2\epsilon}{\nu}\frac{2}{2J+1} \times \Bigg\{ \left(1 - \frac{m_l^2 + q_\mu^2}{4\epsilon\nu}\right) \Bigg[\sum_{J\geq 0}^{\text{even}} |\langle J_f \| \hat{M}_J \| J_i \rangle|^2 \\
&+ \frac{1}{2}\sum_{J\geq 1}^{\text{odd}} \left(|\langle J_f \| \hat{T}^{mag}_J \| J_i \rangle|^2 + |\langle J_f \| \hat{T}^{el5}_J \| J_i \rangle|^2 \right) \Bigg] + \left(\frac{m_l^2 + q_\mu^2}{4\epsilon\nu}\right) \times \\
&\Bigg\{ \sum_{J\geq 1}^{\text{odd}} \Bigg[\left(1 - \frac{m_l^2}{2|\vec{q}|^2}\right) \left(|\langle J_f \| \hat{T}^{mag}_J \| J_i \rangle|^2 + |\langle J_f \| \hat{T}^{el5}_J \| J_i \rangle|^2 \right) \\
&+ \frac{m_l^2}{|\vec{q}|^2} |\langle J_f \| \left(1 - \frac{|\vec{q}|^2}{m_\pi^2 + q_\mu^2}\right) \hat{l}^5_J \| J_i \rangle|^2 \Bigg] \Bigg\} \mp \left[\left(\frac{m_l^2 + q_\mu^2}{4\epsilon\nu}\right)^{\frac{1}{2}} (m_l^2 + |\vec{q}|^2)^{\frac{1}{2}} - \frac{m_l^2}{2\epsilon} \right] \frac{1}{|\vec{q}|} \times \\
&\sum_{J\geq 1}^{\text{odd}} 2\text{Re}\langle J_f \| \hat{T}^{mag}_J \| J_i \rangle \langle J_f \| \hat{T}^{el5}_J \| J_i \rangle^* \Bigg\} \times \\
&\frac{[|\vec{q}|^2 + M_2^2]^{\frac{1}{2}}}{M_1} \left[1 + \frac{\nu}{M_1} \left(1 - \frac{\epsilon(2\epsilon\nu - m_l^2 - q_\mu^2)}{2\nu(\epsilon^2 - m_l^2)} \right) \right]^{-1}
\end{aligned}$$

For neutral current scattering, $m_l{}^2 = 0$.

The charged muon capture rate is given by

$$\omega_{fi} = \frac{G^2\nu^2}{2\pi}|\phi_{1s}|^2_{av}\frac{4\pi}{2J_i+1}\times$$
$$\left\{\left\{\sum_{J=0}^{\infty}|\langle J_f\|\hat{\mathcal{M}}_J - \hat{\mathcal{L}}_J + \frac{(q-q_0)}{(m_\pi^2+q^2)}\left(q\,\hat{l}^5_J - q_0\,\hat{m}^5_J\right)\|J_i\rangle|^2\right\}\right.$$
$$\left.+\left\{\sum_{J=1}^{\infty}|\langle J_f\|\hat{\mathcal{T}}^{mag}_J - \hat{\mathcal{T}}^{el}_J\|J_i\rangle|^2\right\}\right\}$$

The β - decay rate, in the long - λ limit, is given in ref. 9.

The total cross-section is obtained by integration over the possible solid angles. It is convenient to integrate over q^2, where q is related to θ by

$$q_\mu{}^2 = 2\epsilon\nu\left(1-\frac{\sqrt{\epsilon^2-m_l^2}}{\epsilon}\cos\theta\right) - m_l^2.$$

Combining with $\epsilon^2 = (\nu+M_1)^2 - 2(\nu+M_1)\sqrt{M_2^2+|\vec{q}|^2} + (M_2^2+|\vec{q}|^2)$ we can obtain the correct limits of integration.

We have carried out some checks of our derived results against the conventional results.[9] We first checked that the relativistic Hartree formula reduces to the conventional formula in the ERL limit, where terms proportional to m_l disappear. We next checked the nonrelativistic limit of the cross-sections, by taking the nonrelativistic limit of the wavefunctions,

$$\psi \equiv \begin{pmatrix} \chi(r) \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{2M}\chi(r) \end{pmatrix},$$

where $\chi(r)$ is the upper radial component. We took matrix elements of the relativistic multipole operators with the nonrelativistic limit of the wavefunctions, retaining terms to order (1/M), and verified that this gave exact agreement with the result obtained by substituting the conventional non- relativistic multipoles into the non-relativistic formulas.[1]

Using the expressions for the cross-sections derived above, we have computed cross-sections and rates for various processes involving the $^{17}\mathrm{F} - ^{17}\mathrm{O}$ isodoublet system, and compared with the corresponding nonrelativistic limits. We treat the A = 17 system as a valence $1d_{\frac{5}{2}}$ neutron orbiting an inert ^{16}O core. Fig. 1 shows the differential cross-sections for the charge changing process $\nu_e + ^{17}\mathrm{O} \rightarrow e^- + ^{17}\mathrm{F}$ as well as the neutral current scattering process $\nu + ^{17}\mathrm{O} \rightarrow \nu + ^{17}\mathrm{O}$. The T = 0 multipoles from the core (^{16}O) contribute to the neutral ν - scattering process. Since in principle there are no inherent limitations as to the kinematic range for the probes, we have examined the high - q behavior of the differential cross - sections, pushing to q = 1.2 GeV. We observe interesting diffraction structure in the high - q region, though the cross-section is too small for experimental verification in the near future.

The integrated cross-sections are shown in Fig. 2 together with the corresponding non-relativistic limits, where only the upper components of the wave-functions are used and terms are retained only to order (1/M). As the curves approach their asymptotic limits, the relativistic correction is seen to be around 5 % for ν - scattering and 9 % for the charge changing reaction.

Just to see what the relativistic corrections are, we have also studied the 3 - body system $^3\mathrm{H} - ^3\mathrm{He}$ with our formalism, using a simple model for ^{3}He of a $1s_{\frac{1}{2}}$ neutron hole in a ^{4}He core The upper part of Fig. 3 shows the integrated cross-sections for the ν - and ν' - scattering from ^{3}He. The lower portion shows the total cross-sections for the anti-neutrino charge changing reactions $\bar{\nu}_l + ^3\mathrm{He} \rightarrow ^3\mathrm{H} + l^+$, where $l = e$ and μ. Here the comparison is with results previously obtained with the nonrelativistic harmonic oscillator formalism.[9] The magnitude of the relativistic corrections are comparable to the A=17 system.

We now turn to the conventional weak processes (β - decay and μ - capture). Table 1 shows the β - decay rates ($^3\mathrm{H} \rightarrow ^3\mathrm{He} + e^- + \bar{\nu}_{e^-}, ^{17}\mathrm{F} \rightarrow ^{17}\mathrm{O} + e^+ + \nu_e$). For both nuclei, the relativistic result is compared with experimental values[10,11] and with the nonrelativistic limit of the new results as defined above. The discrepancies with the nonrelativistic results is less than 1 %. However, the approximations inherent in our calculations do not yet permit quantitative explanation of the relativistic effects. It happens that the agreement with experimental values is within 1.5 % for the A = 3 system, and 12 % for the A = 17 system. Table 1 also shows the rates for μ - capture on helium ($\mu^- +^3\text{He} \rightarrow^3 \text{H} + \nu_\mu$). The new result differs from the nonrelativistic result by 5 % , and lies on the lower end of the experimental error-bar.[13]

In summary, using the QHD-formalism we have calculated nuclear semileptonic processes in a consistent relativistic framework: relativistic corrections are thus explicitly summed to all orders. We observe that the agreement with the non-relativistic limit for the rates and integrated cross-sections for the processes is within 9 %. In order to probe the quark stucture of nuclei, one must carry the current hadronic analysis out to extreme kinematics. With no inherent limitations to the kinematic range of probes in our formalism, we have provided predictions for a wide range of q^2 for the exotic processes (neutrino reactions and scattering). High priorities for improvements of our analysis include extensions to more general nuclear configurations and a relativistic treatment for the center-of-mass correction factor. Since we have a consistent field-theoretical framework, we can now systematically examine corrections to this analysis.

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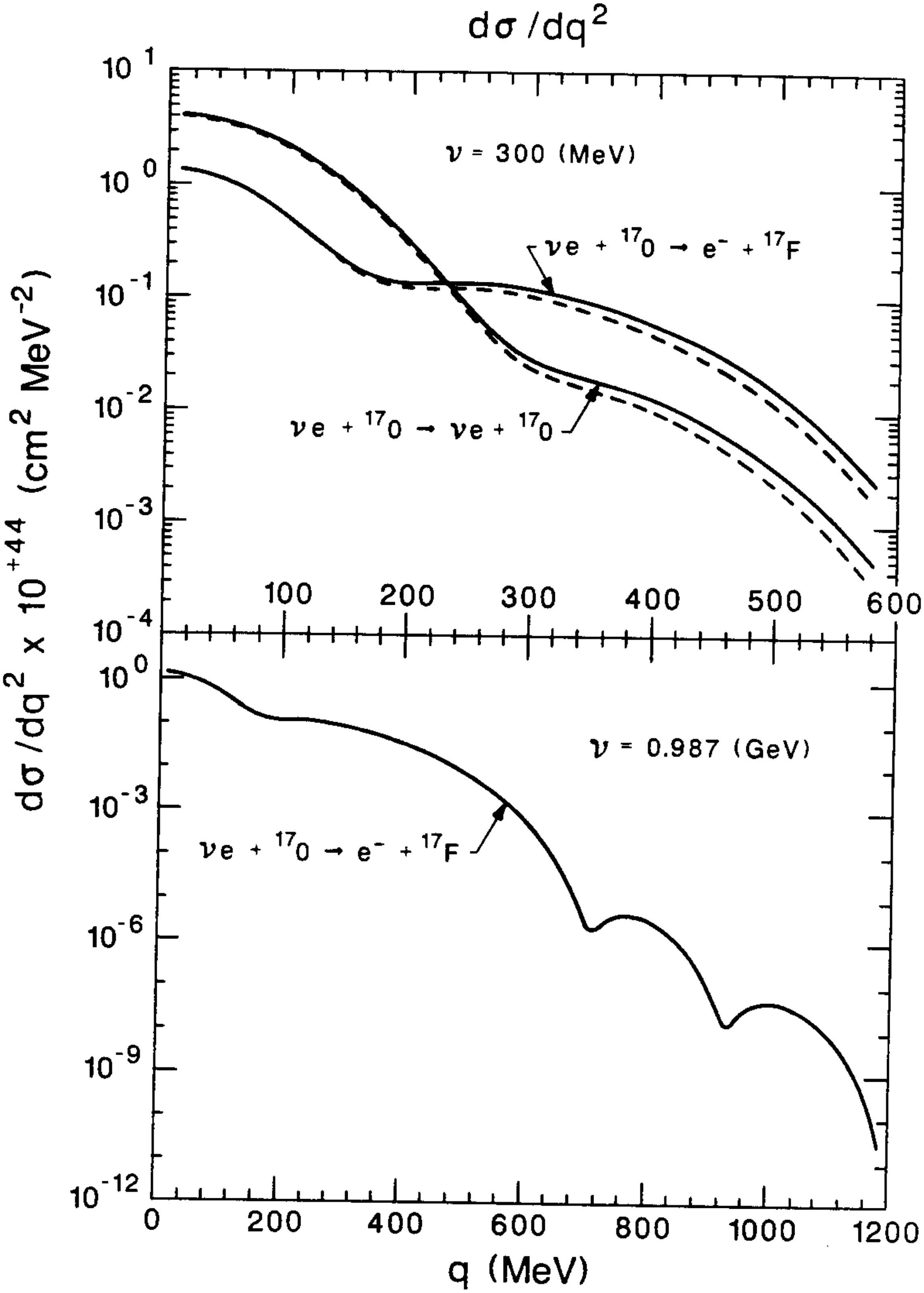
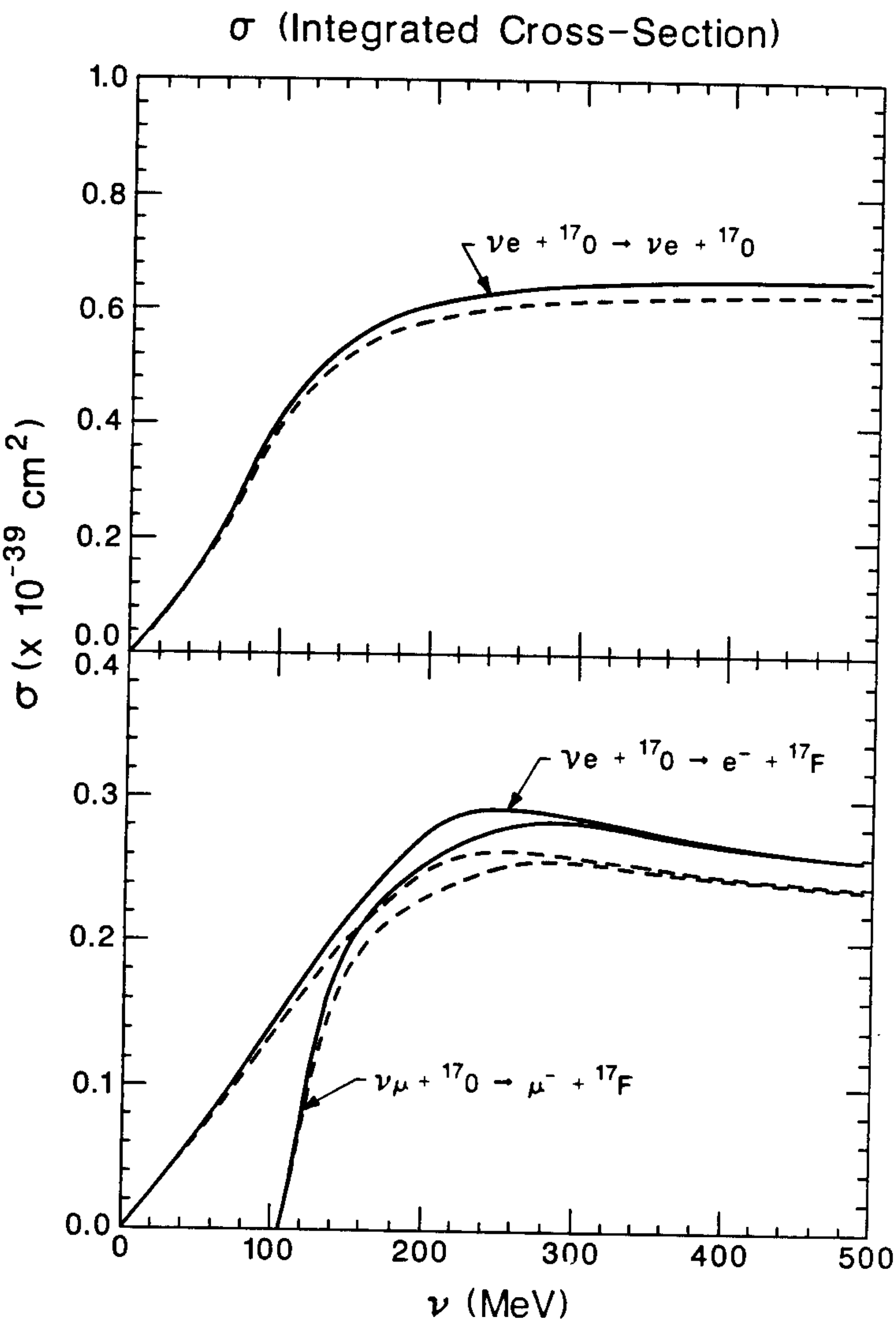
Table 1 ($\beta-$ Decay and $\mu-$Capture Rates in sec^{-1})

Process	Rel. Hartree	Nonrel. Limit	Experiment
$^{3}\mathrm{H} \rightarrow ^{3}\mathrm{He} + e^{-} + \nu_{e^{-}}$	1.815×10^{-9}	1.818×10^{-9}	$(1.7906 \pm 0.0067) \times 10^{-9}$
$^{17}\mathrm{F} \rightarrow ^{17}\mathrm{O} + e^{+} + \nu_{e}$	1.223×10^{-2}	1.228×10^{-2}	1.075×10^{-2}
$\mu^{-} + ^{3}\mathrm{He} \rightarrow ^{3}\mathrm{H} + \nu_{\mu}$	1458	1378	1505 ± 46

FIGURE CAPTIONS

1. Solid curve: relativistic Hartree result. Dashed curve: nonrelativistic limit.
2. Solid curve: relatvistic Hartree result. Dashed curve: nonrelativistic limit
3. Solid curve: relativistic Hartree result. Dashed curve: nonrelativistic harmonic oscillator result


dσ /dq^2
ν = 300 (MeV)
νe + ^{17}O → e$^-$ + ^{17}F
νe + ^{17}O → νe + ^{17}O
ν = 0.987 (GeV)
νe + ^{17}O → e$^-$ + ^{17}F
dσ /dq^2 x 10^{+44} (cm^2 MeV^{-2})
q (MeV)



σ (Integrated Cross-Section)
1.0
0.8
0.6
0.4
0.2
0.0
0.4
0.3
0.2
0.1
0.0
σ (x 10^-39 cm^2)
νe + 17O → νe + 17O
νe + 17O → e⁻ + 17F
νμ + 17O → μ⁻ + 17F
0
100
200
300
400
500
ν (MeV)

σ (Integrated Cross-Section)